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SUBSONIC RAREFIED GAS FLOW OVER A RACK OF FLAT TRANSVERSE PLATES

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Parameters of a rarefied gas flow through a rack of flat plates aligned across the flow are studied by means of the joint numerical solution of the Boltzmann and Navier–Stokes equations. A subsonic flow regime is considered. The changes in flow characteristics are calculated as functions of the free-stream velocity and plate temperature.

Key words: *numerical calculation, Boltzmann equation, Navier–Stokes equations, transverse plate, rack, permeable target, transonic flow, heat transfer.*

Introduction. Akin'shin et al. [1] performed a pioneering experimental study of a rarefied gas flow in a capillary sieve in the range of Knudsen numbers $\mathrm{Kn} = 10^{-4}$–10^{-1} at different temperatures for several inert gases and determined the dependence of the flow rate on the flow geometry and the character of interaction between molecules and capillary walls. A supersonic flow with Mach numbers $\mathrm{M} = 2$–3 and a Reynolds number $\mathrm{Re} = 10^6$ over perforated screens was studied in [2] by analyzing the effect of injection into the base region on the flow structure. Theoretical results based on the presentation of the perforated screen as a discontinuity surface with relations that take into account the flow structure inside the perforation and the mechanisms of interaction between the gas and the walls were published in [3], and this analytical approach was further developed in [4, 5].

The results of numerical experiments performed by means of direct statistical modeling of one-dimensional rarefied gas flows through a permeable flat surface can be found in [6]. Steady-state flows in the range of Mach numbers $\mathrm{M} = 3$–10 with different temperatures of the target and accommodation numbers were studied under the assumption that the gas molecule passes through the surface without interacting with the latter with a probability P and becomes scattered with a probability $1-P$. In particular, the laws of variation of flow parameters in the "shock-disturbance front" formed ahead of the target were determined, and the difference of this front from the shock-wave structure was found.

The necessity of studying subsonic rarefied gas flows is inspired by their importance for practical purposes (motion of the gas in porous solids and capillary membranes in devices for gas separation and cooling). The flow characteristics are determined by interaction of flows around the target elements. These are normally three-dimensional structures consisting of cylindrical channels or a set of finite-length plates aligned along or across the flow.

It seems of interest to consider systems designed for controlling (generating) flows with prescribed properties without any changes in geometry, by varying the temperature or gas-flow parameters on the wetted surfaces. Such systems are analyzed in the present paper.

The numerical solution of the Boltzmann equation, which describes the rarefied nonequilibrium gas dynamics, requires a large volume of computations, because the distribution function to be determined includes a three-dimensional space of velocities in addition to spatial variables. Two-dimensional formulations can be currently used for numerical simulations of rarefied gas flows. Even in comparatively simple cases, however, the number of nodes of the distribution function reaches 5–10 million, and solving the kinetic equation in the entire domain is extremely

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labor-consuming, especially if the flow occupies a large space and the steady state is reached rather slowly. These features are typical of subsonic regimes, where a large computational domain is needed for the boundary conditions to be adequately imposed. To overcome these difficulties, Popov and Tcheremissine [7, 8] proposed and tested a method that allows calculating essentially nonequilibrium flow regions by the unsteady Boltzmann equation and solving the unsteady Navier–Stokes equations in regions where the state of the gas is close to thermodynamic equilibrium. The solutions are matched at each time step on a previously defined contour by using the Enskog–Chapman function; the requirement of continuity of the mass, momentum, and energy fluxes are satisfied thereby.

The Boltzmann equation is solved by the finite-difference method on fixed space and velocity grids. Rigorous satisfaction of conservation laws and vanishing of the collision integral on the locally Maxwellian distribution function are reached by using the projection method [9, 10]. The collision integral is calculated on a cubature grid, which is common for all spatial nodes, in the space of velocities; the Monte Carlo method is not used. The collision integral can be calculated for a large number of intermolecular potentials, because the deflection angle is found by solving an appropriate scattering problem [11].

The effectiveness of using the matching procedure in studying complex flows was demonstrated in [12, 13]. Nonmonotonicity at the matching boundary is undesirable and can be caused by two reasons: 1) incorrect choice of the domain of integration of the Boltzmann equation in the space of velocities or placing the matching boundary in an essentially nonequilibrium region (which can be easily corrected by choosing appropriate parameters); 2) different descriptions of the shock-wave structure by the kinetic model and the continuum model for $\mathrm{M} > 1.5$: in this case, the shape of strong shock waves at the matching boundary is distorted, though the velocities and the values of the macroscopic quantities behind the shock wave are determined correctly (these discrepancies cannot be avoided).

In addition to studying comparatively simple cases of the rarefied gas flow with prospects of considering more complicated systems, it is also possible to investigate a number of the classical problems of gas dynamics, for instance, the shock-wave structure and the interaction of shock waves with each other and with surfaces and boundary and shear layers. These problems include the problem of the structure of local supersonic regions in transonic flows [14].

The present work deals with an unsteady process of interaction of a subsonic rarefied gas flow with a rack instantaneously inserted into the flow. Overlapping of shock waves reflected from the plates leads to formation of a plane shock wave propagating upstream. Mixing of jet flows in the rack gaps results in formation of a steady flow near the rack, which is consistent with the flow behind a detached shock wave. Some features of formation of local supersonic regions at $\mathrm{Kn} = 0.10$–0.05 are examined, which were not considered previously.

Formulation of the Problem. A periodic rack is inserted into a homogeneous continuous rarefied gas flow. The rack consists of infinitely thin flat plates of length L, and the distance between these plates is l. The plates are aligned in the plane oriented across the flow. The length of periodicity is $L + l$ ($L/l = S$). If the plates in the rack are located at a substantial distance from each other, then $S \to 0$, which describes the flow past individual plates. For $S \to \infty$, the incoming flow interacts with a plane possessing extremely narrow slots. The free-stream parameters are the density n_0, the temperature T_0, the velocity $u_0 = \mathrm{M}_0\sqrt{\gamma T_0}$, and the ratio of specific heats $\gamma = 5/3$. The length and time units are the mean free path and time of molecules in the incoming flow. The density and temperature are normalized to their free-stream values ($n_0 = 1$ and $T_0 = 1$).

These problems have a large number of parameters; hence, the resultant flows are rather versatile. The calculations were performed for fixed geometric parameters of the rack $L = 10$ and $S = 0.77$; the varied parameters were the subsonic free-stream velocity and the temperature conditions on the plates.

In the domain where the Boltzmann equation was solved (shown by the rectangle bounded by the dashed curve in Fig. 1), we used a power potential of molecular interaction with a power index of 1/12. The gas–surface interaction follows the diffuse reflection law at this surface temperature. The viscosity and thermal conductivity in the Navier–Stokes equations were assumed to be proportional to $T^{2/3}$.

As the flow is periodic with respect to Y, the calculations were performed in the domain $0 < Y < Y_g$. Conditions of symmetry were set for $Y = Y_g$ and on the plane $Y = 0$ passing through the plate center. The Boltzmann equation was solved on a 60×20 spatially uniform grid, and approximately 4000 nodes were set in the space of velocities. The same uniform coordinate grid with the number of nodes from 350×23 to 350×45 in the x and y directions, respectively, depending on a particular variant of computations, was used in the Navier–Stokes equations.

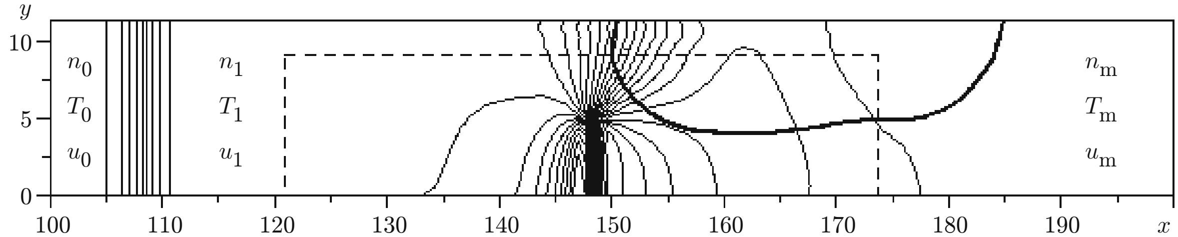

Fig. 1. Flow structure near the rack for $M_0 = 0.95$, $T_w = 1$, $L = 10$, and $S = 0.77$: the dashed curve is the boundary of the domain where the Boltzmann equation was solved; the solid curve is the boundary of the supersonic flow region.

The proposed computational scheme allows not only determining the general characteristics of the flow but also analyzing the processes on the rack at the kinetic level. This makes it possible to calculate the drag force, friction, heat flux, temperature jump, and slip velocity on the plate.

The drag coefficient C_x, friction coefficient C_f, and heat-transfer coefficient C_q are determined as follows:

$$C_x = \frac{2F_x}{n_0 m u_0^2 L}, \qquad C_f = \frac{2F_y}{n_0 m u_0^2 L}, \qquad C_q = \frac{Q}{n_0 m u_0^3 L},$$

$$F_x = \int p_{xx}^{(1)}\,dy - \int p_{xx}^{(2)}\,dy, \quad F_y = \int p_{xy}^{(1)}\,dy + \int p_{xy}^{(2)}\,dy, \quad Q = \int q^{(1)}\,dy - \int q^{(2)}\,dy.$$

Here F_y is the friction force acting on the upper part of the plate, m is the mass of the molecule, q is the heat flux, and p_{xx} and p_{xy} are the friction-stress components; integration is performed for $Y > 0$ over the surface turned toward the incoming flow for quantities marked by the superscript (1) and over the backward surface for quantities marked by the superscript (2). The values of p_{xx}, p_{xy}, and q are found by integration in the space of velocities:

$$p_{xx}^{(1,2)} = \int m\xi_x^2 f_s\,d\boldsymbol{\xi}, \qquad p_{xy}^{(1,2)} = \pm \int m\xi_x\xi_y f_s\,d\boldsymbol{\xi}, \qquad q^{(1,2)} = \pm\frac{1}{2}\int \xi_x m\xi^2 f_s\,d\boldsymbol{\xi}$$

[ξ_x and ξ_y are the velocities of molecules in the x and y directions, respectively, and ξ is the absolute value of velocity; the signs plus and minus correspond to the superscripts (1) and (2)].

Let $\boldsymbol{b}$ be the outward normal to the surface. The distribution function f_s on the plate is determined as the sum

$$f_s = f_i^* + f_w^*,$$

$$f_i^* = \begin{cases} f_i, & (\boldsymbol{\xi}, \boldsymbol{b}) < 0, \\ 0, & (\boldsymbol{\xi}, \boldsymbol{b}) \geqslant 0, \end{cases} \qquad f_w^* = \begin{cases} f_w, & (\boldsymbol{\xi}, \boldsymbol{b}) > 0, \\ 0, & (\boldsymbol{\xi}, \boldsymbol{b}) \leqslant 0. \end{cases}$$

This formulation shows that the part of the distribution of velocities directed toward the plate is taken from f_i, and the part of the distribution with the opposite direction of velocities is taken from f_w. The function f_i is determined by solving the problem numerically, and the function f_w is found by the formula

$$f_w = n_w\Big(\frac{m}{2\pi k T_w}\Big)^{3/2} \exp\Big(-\frac{m\xi^2}{2\pi k T_w}\Big).$$

The plate temperature T_w is assumed to be given, and n_w is found from the no-slip condition

$$\int\limits_{(\boldsymbol{\xi},\boldsymbol{b})<0} \xi_x f_i\,d\boldsymbol{\xi} = \int\limits_{(\boldsymbol{\xi},\boldsymbol{b})>0} \xi_x f_w\,d\boldsymbol{\xi}.$$

The temperature jump ΔT and the slip velocity v_s are described by the relations

$$\Delta T^{(1,2)} = \frac{m}{3n_s k}\int \xi^2 f_s\,d\boldsymbol{\xi} - T_w, \qquad v_s^{(1,2)} = \frac{1}{n_s}\int \xi_y f_s\,d\boldsymbol{\xi}, \qquad n_s^{(1,2)} = \int f_s\,d\boldsymbol{\xi}.$$

The quantities on the plate are normalized as follows: $p_{xx}^{(1,2)}/(mn_0u_0^2)$, $p_{xy}^{(1,2)}/(mn_0u_0^2)$, $2q^{(1,2)}/(mn_0u_0^3)$, $\Delta T^{(1,2)}/T_0$, and $v_s^{(1,2)}/u_0$.

Basic Results. In the present paper, we consider the flow chosen in the following manner. As in studying similar supersonic flows [12], the plate length L is fixed and equals 10 mean free paths in the incoming flow. The parameter S is chosen so that the changes in plate temperature exert a significant effect on the resultant flow near the rack and behind it. If the gaps between the plates are greater than the plate size by a factor of 2–3, the conditions on the surface have a minor effect on the flow. As the gap size is reduced to fractions of the plate length, the gas temperature behind the plate is approximately equal to the plate temperature, and the density can be determined if the pressures are approximately identical. Therefore, the interval of the values of S of the order of unity (in the present work, $S = 0.77$) is of greatest interest from the viewpoint of estimating the changes in flow parameters induced by varying the conditions on the rack. The flow is essentially two-dimensional near the plates only and rapidly become close to one-dimensional at a distance of several hundreds of the mean free paths from the rack. This fact facilitates the study of essentially subsonic regimes that require the boundary conditions to be set at a large distance. For $\mathrm{M}_0 = 2.5$ [12], a supersonic flow with a local Mach number $\mathrm{M}_\mathrm{m} = 1.5$ is established behind the rack. As M_0 decreases, the configuration of the supersonic region changes, and local supersonic regions in the gaps of the rack are formed at $\mathrm{M}_0 \approx 1.1$. With a further decrease in free-stream velocity, the supersonic regions become smaller. The case examined in the present paper ($\mathrm{M}_0 = 0.6$) is the intermediate regime between completely subsonic flows and flows with local supersonic regions.

The overall pattern of the flow with a local supersonic region is shown in Fig. 1 together with the calculated results for $\mathrm{M}_0 = 0.95$ and $S = 0.77$. The density field for $t = 60$ is shown, which corresponds to the beginning of the process. Interaction between the incoming flow ($n_0 = 1$, $T_0 = 1$, and $u_0 = 1.23$) and the rack generates a reflected shock wave. This wave propagates upstream with a relative velocity of 0.66 and leaves a gas flow ($n_1 = 1.65$, $T_1 = 1.47$, $u_1 = 0.48$, and $\mathrm{M}_1 = 0.31$) behind it. Passing through the rack gaps, the gas flow becomes accelerated and forms a supersonic region (solid curve in Fig. 1). After a certain time, the shock wave goes away in the upstream direction, and the flow near the plate and further downstream becomes almost steady; the mean parameters of this flow along the y coordinate are $n_\mathrm{m} = 0.75$, $T_\mathrm{m} = 1.09$, $u_\mathrm{m} = 1.06$, and $\mathrm{M}_\mathrm{m} = 0.81$. The shape of the supersonic region is usually close to that shown in Fig. 1.

The evolution of the flows under consideration is described by several characteristic times. The first of them corresponds to formation of the reflected shock wave and formation of the flows in the rack gaps in the first approximation. These processes occur during the time $t = 50$–100 and correspond to the state of the flow shown in Fig. 1. During the time $t \approx 200$–300, the density and temperature fields near the rack become stabilized, but the flow rate determined by the integral of the product of density and streamwise velocity differs from the constant value by 3–5%. This difference disappears by the time $t \approx 1000$. Note that the experiments showed that the solutions obtained are independent of the place of the downstream boundary if the latter was located at a distance of at least 150–200 units. The so-called soft conditions (absence of flows) were imposed on this boundary. Free-stream conditions were maintained on the left boundary. When the shock wave approached this boundary, the latter was shifted by 10 units toward the rack, and the values of n_1, u_1, and T_1 were set there.

Most results discussed in the paper refer to the variant of the flow with $\mathrm{M}_0 = 0.6$ and $T_w = 1$. The distributions established near the rack by the time $t = 180$ slowly evolve to the steady-state solution, changing within several percent. The fields of density, temperature, and streamwise velocity are plotted in Fig. 2. In addition, Fig. 2 shows the region where the Boltzmann kinetic equation was solved ($dx = 0.85$, $dy = 0.5$, and $dt = 0.1$). At the moment illustrated in Fig. 2, the shock wave has already gone far to the left and formed a subsonic flow with parameters $n_1 = 1.32$, $T_1 = 1.21$, $u_1 = 0.39$, and $\mathrm{M}_1 = 0.27$, which is incoming onto the rack.

As the plate is assumed to be very thin and to be located between the grid nodes, it is in the middle of the linear profile. For correct visualization, the isolines located at a distance $dx/2$ ahead of the plate and behind it have to be shifted right up to the plate.

Let us determine the specific flow rate R as the ratio of the sum of the products $n_i u_i$ over all computational points in the y direction to the number of these points. Then, the flow rate is $R_0 = 0.77$ in the entrance cross section and acquires the value $R_1 = 0.51$ behind the shock wave. For convenience, we introduce the flow rate as $R_g = R_0(L + l)/l$. In the case considered, it equals 0.44, i.e., is commensurable with R_1. The mean density, temperature, and streamwise velocity behind the rack in the cross section x_m are $n_\mathrm{m} = 0.80$, $T_\mathrm{m} = 1.08$, $u_\mathrm{m} = 0.65$, and $\mathrm{M}_\mathrm{m} = 0.48$. The close values of the flow rates $R_\mathrm{m} = 0.52$ and R_1 testify that the flow regime is almost steady.

The distributions of the main flow parameters near the plates are plotted in Figs. 3 and 4.

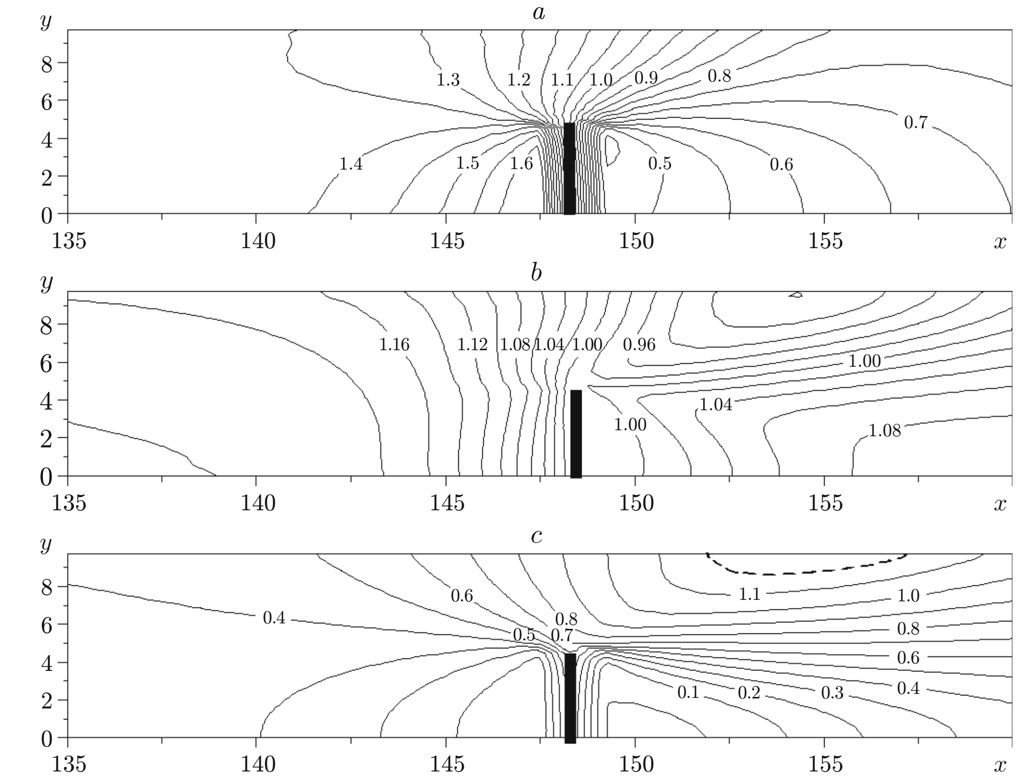

Fig. 2. Flow structure near the rack for $M_0 = 0.6$, $T_w = 1$, $L = 10$, and $S = 0.77$: (a) density; (b) temperature; (c) streamwise velocity; the dashed curve is the boundary of the supersonic flow region.

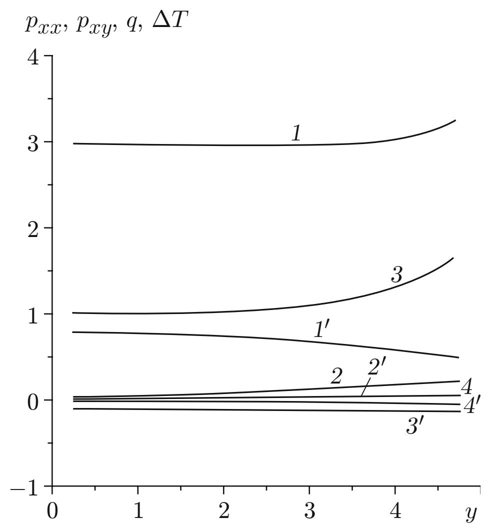

Fig. 3

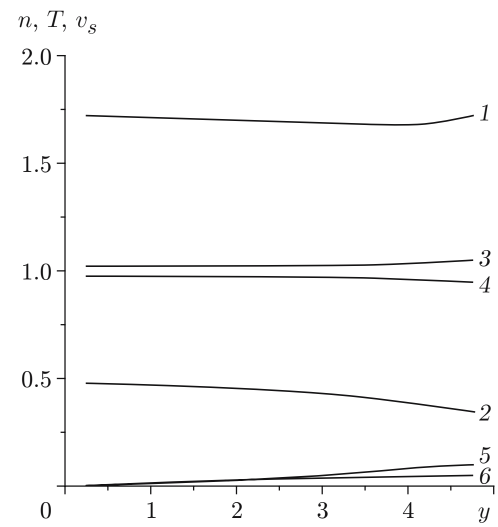

Fig. 4

Fig. 3. Distributions of parameters along the plate for the flow whose structure is shown in Fig. 2: curves 1–4 are the parameters on the plate surface turned toward the incoming flow; curves 1′–4′ show the same parameters on the opposite surface; the parameters shown are the momentum flux (1 and 1′), friction forces (2 and 2′), energy fluxes (3 and 3′), and temperature jumps (4 and 4′).

Fig. 4. Distributions of densities (1 and 2), temperatures (3 and 4), and shear velocities (5 and 6) at a distance of 0.42 from the plate surface for the flow whose structure is shown in Fig. 2 ($C_x = 2.33$, $C_q = 1.04$, and $C_f = 0.13$).

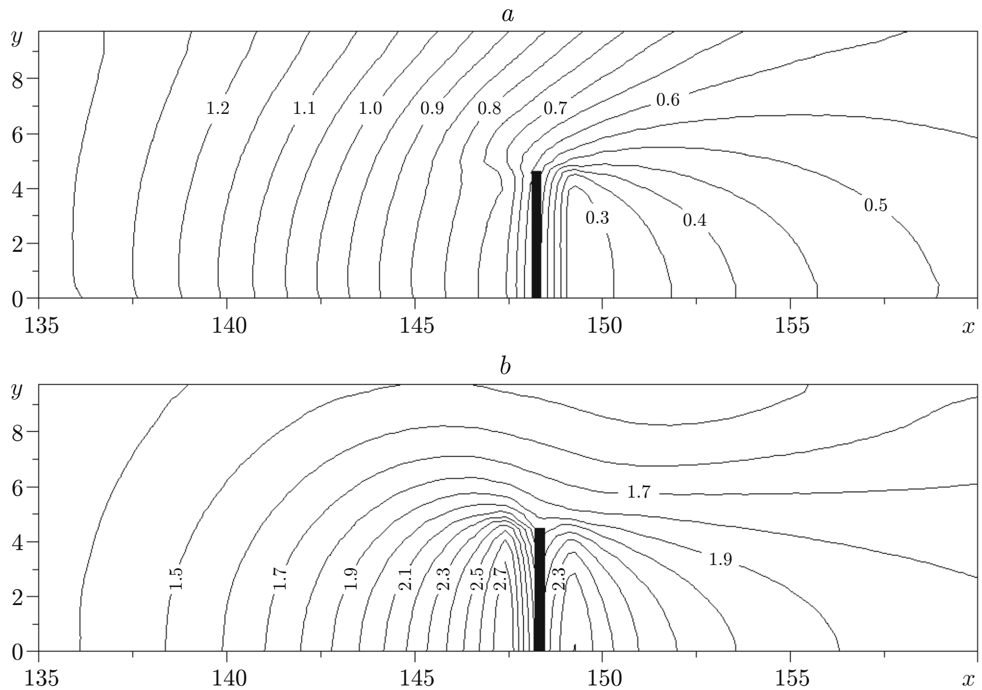

Fig. 5. Flow structure near the rack for $T_w = 3.5$: (a) density; (b) temperature.

The effect of the rack surface temperature on the flow structure is illustrated in Fig. 5, which shows the fields of density and temperature corresponding to the flow shown in Fig. 2, but for higher temperatures on both plate surfaces ($T_w = 3.5$). A change in the conditions of shock-wave reflection leads to changes in shock-wave velocity and parameters behind it: $n_1 = 1.41$, $T_1 = 1.28$, $u_1 = 0.29$, and $\mathrm{M}_1 = 0.19$. The flow rate $R_1 = 0.41$ is compared with the geometric flow rate R_g. A flow with parameters $n_\mathrm{m} = 0.55$, $T_\mathrm{m} = 1.77$, $u_\mathrm{m} = 0.77$, and $\mathrm{M}_\mathrm{m} = 0.45$ is formed behind the rack. It should be noted that flow stabilization behind the rack in the case considered occurs at a much greater distance that in the case of a cold rack. The temperature jump is approximately 0.5 on the plate surface turned toward the flow and is close to unity on the opposite surface; the slip velocity ranges from 0.05 to 0.10. The main characteristics of interaction take the values $C_x = 2.51$, $C_q = -6.64$, and $C_f = 0.097$.

Conclusions. In the subsonic flows considered, the intensity of the detached shock wave and the throughput of the rack are primarily determined by the rack geometry, namely, its porousness. An analysis of results for flows with different plate temperatures T_w shows that the flow rate can be controlled within tens of percent by changing the temperature from T_0 to $3.5T_0$ (mainly by changing the conditions of flow reflection from the rack in the initial period of shock-wave formation). The temperature and density of the flow passing through the rack significantly depend on the temperature conditions on the plate surfaces. Thus, if the plate surface temperature is increased by a factor of 3.5, the flow behind the plate is heated by a factor of 1.7, and the density decreases accordingly. The flow parameters in the vicinity of the plates change even more significantly. Regions of rarefaction and cooling (see Fig. 5) are formed instead of regions of compression and heating (see Fig. 2). The fluxes of momentum and energy impinging on the rack elements experience not only quantitative but also qualitative changes.

For $\mathrm{M}_0 < 0.6$ ($L = 10$ and $S = 0.77$), the flow behind the rack is subsonic everywhere. As the free-stream velocity increases, local supersonic regions are formed in the gaps of the rack, whereas the flow remains subsonic in the shaded regions behind the plates. For $\mathrm{M}_0 > 1.1$, the supersonic regions overlap and form a common supersonic flow at a certain distance from the rack [12]. Local supersonic regions in transonic continuum flows are usually closed by shock waves. The width of weak shock waves in a rarefied gas is several tens of the mean free paths. In the flows considered here, the size of the local supersonic region is comparable with the width of the shock wave formed behind this region, and their structures overlap.

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